Survey of India.

PROFESSIONAL PAPER—NO. 1.

(*SECOND EDITION*).

ON THE PROJECTION FOR A MAP OF INDIA AND ADJACENT COUNTRIES

On the Scale of 1:1,000,000

BY

COLONEL ST. G. C. GORE, C.S.I., R.E.

PREPARED UNDER THE DIRECTION OF

COLONEL ST. G. C. GORE, C.S.I., R.E.,
SURVEYOR GENERAL OF INDIA.

Dehra Dun:
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PREFACE TO THE SECOND EDITION.

When the first edition of this pamphlet was published, very little attention had been paid to the question of projections for a considerable number of years. The projection then decided on, *viz.*, the ordinary secant conical one, was and is considered amply good for the requirements of the map.

The issue of the pamphlet drew attention to the subject of projections and criticisms adverse to the adopted projection were made. These criticisms valid as they are from a theoretical point of view lose some of their force in view of the above statement that the projection was good enough for the purpose for which it was adopted. But as the criticisms have been made, rather than burden the map with any breath of suspicion, I have determined to alter the projection to the one now given.

For the mathematical solution of the problem of the projection, I am indebted to Mr. J. Eccles, M. A., Officiating Superintendent Trigonometrical Surveys.

The sheets already drawn or nearly so on the old projection, will be issued as a preliminary edition and a new edition brought out as early as possible.

ST. G. C. GORE, COLONEL, R.E.

ON THE PROJECTION

FOR A MAP

OF

INDIA AND ADJACENT COUNTRIES

ON THE SCALE OF 1:1,000,000.

The word 'map' is used rather than 'atlas' to denote a considerable number of detached sheets because the term 'atlas of India' is already in use and it was feared that the use of the word 'atlas' might lead to confusion.

The extent of country which it is proposed that the new map should embrace, reaches longitudinally from the western frontier of Persia to the east of China, that is, from 44° to 124° east longitude, and in latitude from the south of Ceylon to the Oxus or from 4° to 40° north latitude.

The scale of the map, $\frac{1}{1,000,000}$, has been chosen to fall in with the views expressed by the International Geographical Congress, who have proposed that a map of the world be undertaken on that scale.

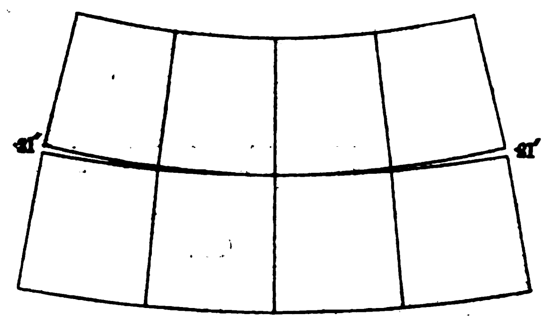
A general map of India and Adjacent Countries on the scale of 16 miles to one inch has long been a desideratum, and the scale of the proposed map, *viz.*, $\frac{1}{1,000,000}$, being equivalent to one of 15·78 miles to one inch, is so very close to that of the map required that it may well be substituted for it.

The system of projection to be proposed by the International Geographical Congress has not yet been decided upon, and as it is inadvisable to delay the commencement of the map, it has become necessary to select a suitable projection.

2. Broadly speaking there are two systems of projection which may be adopted for such a map; one is that in which each sheet is projected on its own central meridian and the other is to have one projection for the whole map.

The first method has the great advantage that by its use each sheet can be made practically true to scale and free from distortion. Examples of projections suitable for this method are the ordinary Survey of India projection used for standard sheets and the Rectangular Tangential projection of Sir H. James, employed by the Home War Office. Over moderate areas these two projections may be regarded as identical.

The disadvantage of this method is that it is impossible to join any number of sheets together to make a large map. Thus in the Rectangular Tangential projection if it is attempted to join up 8 sheets $4^\circ \times 5^\circ$, of a $\frac{1}{1,000,000}$ map together on their central meridian as in the sketch, the two belts will separate at each end on the central parallel to the extent of ·21 inch. The Survey graticule gives about the same amount of misfit.



3. As it has been considered essential in the case of the present map that all the sheets should fit together exactly to form one map, the second of the above-mentioned methods must be adopted, that is there must be one projection for the whole map.

As the area to be included is a very large portion of the globe, embracing 36° of latitude and 80° of longitude, some form of conical projection is clearly indicated.

4. In order to keep distances along the two co-ordinates true to scale, a system such as Bonne's modification of the conical projection suggests itself; but the area is so extensive that it is found that if this projection be adopted the distortion of the corner sheets becomes very marked.

5. As the map embraces 36° of latitude, a simple conical projection on any central parallel, while meeting satisfactorily the difficulties of the great longitudinal expanse, falsifies the scale very largely in longitude along the north and south limits of the map.

To avoid this, as far as posible, it seems best to adopt a modified secant conical projection somewhat similar to that adopted by Euler in 1777 for his map of Russia. This will minimize the scale error to a considerable extent. A certain amount of error is of course unavoidable; all latitudinal distances, north and south, are correct, while longitudinal distances measured along two parallels are correct, but too short between these parallels and too long beyond their limits north and south. The error in longitude between these is never so much as 2 per cent, that on the extreme northern margin of the map is about $2\frac{1}{4}$ per cent that on the parallel of 8° under 2 per cent, while the error on the extreme southern parallel of 4° is nearly 4 per cent but as the area about this latitude comprises of a small portion of Ceylon and of the Malay Peninsula this error is not important.

6. However the whole amount of error introduced is not of much importance in a map of this class, which is on a scale smaller than that employed in any actual field surveys and it has been decided therefore to adopt the modified secant conical projection, described in detail hereafter.

7. The longitudes hitherto given on all maps published in India have been based on one or other of the old value of that of Madras Observatory, *viz.*, 80° 18′ 30″ for the sheets of the Indian Atlas and 80° 17′ 21″ for standard sheets and general maps.

As the longitude of Madras has now been ascertained within limits of error quite inappreciable on topographical maps and as the map under consideration embraces countries outside India in which our surveys join up with those of other countries, it is advisable to eliminate the error in longitude.

The longitudes therefore of the Map of India and Adjacent Countries will be referrible to the Greenwich meridian, taking that of Madras Observatory as 80° 14′ 54″, the most recent geodetic value.

8. The question of the size and shape of the sheets requires some consideration.

The borders of the sheets of the Indian Atlas are rectangular and projected on one central meridian for the whole atlas. They thus fit accurately together. On the other hand the sheets are very difficult to plot, and most difficult to use for measuring off latitudes and longitudes or for compiling from. Moreover even within the comparatively narrow limits of longitude embraced by the Indian Atlas the flank sheets are much disfigured by the graticule and names being printed at a considerable angle with the margin. This tilting of names is of little importance when the whole extent of the map is contained in one or two sheets as in hand atlases, as the eye follows the curves of the names along the parallels; but in larger scale maps when one sheet only contains a small portion of the map, the disfigurement is very noticeable. In the case of the map under consideration where 76° of longitude are embraced, this defect would be painfully apparent in the outer sheets.

The alternative shape for the sheets is that so long used throughout the Survey Department, where the bounding lines of the sheet are parallels of latitude and longitude. These would give the side margins of the sheet as straight but slightly converging lines, and the top and bottom margins concentric curves. Beyond a very slight increase of trouble in cutting the top and bottom margins whenever it may be necessary to fit two sheets together, this system presents no drawback of any moment, and it is therefore decided to adopt it. The area embraced by each sheet will be 4° of latitude and 4° of longitude.

9. The sheets of India proper will be engraved; those of the outlying portions, *viz*, Persia, Afghanistan &c., will be photozincographed, at least in the first instance.

A border will be designed which will be common to both sets, so as to make the general appearance of the sheets as far as may be similar.

10. It should be noted that in selecting this particular projection for the map, it is in no wise intended to put it forward as an improvement on or as superseding the present Survey Projection as used on standard maps. It is merely considered to fulfil best the conditions which were required to be met in the case of the map under discussion, the chief of which is the correct fitting together of the sheets.

The Modified Secant Conical Projection.

The extent of country projected has for its limits in latitude the parallels 8° and 40°.

The projection is to be such that the meridians shall be represented by straight lines and the parallels by arcs of circles described round a common centre.

The position of this centre and the angle of projection (that is the angle subtended at the centre by an arc of parallel of 1°) are to be determined by the following considerations:—

1° The lengths on the meridians shall be the same as the meridian lengths on the earth.

2° Following the example of Euler, the errors of lengths of the parallels of 8° and 40° shall be equal and each shall be equal to the error of length of the parallel at a certain latitude between these parallels where it is maximum. It is clear that the sign of the latter will differ from the two former.

There will be two parallels on which the errors will be zero so that the projection is a quasi secant conical one though it cannot be illustrated geometrically.

In fig. 1, let P be the pole of the earth

„ Q „ any other point

„ c „ the co-latitude of Q

„ ν „ the normal at Q terminated by the minor axis

„ ρ „ the radius of curvature of the meridian at any point

„ a „ the semi-axis major of the earth

„ e „ the eccentricity

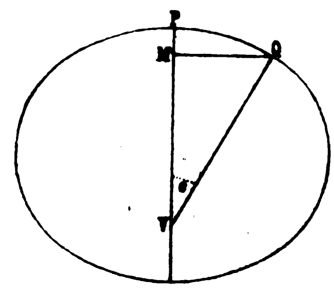


Fig. 1

then

$$\text{arc PQ} = \int_0^c \rho \, dc = \int_0^c \frac{a(1-e^2)}{(1-e^2\cos^2 c)^{\frac{3}{2}}} \, dc$$

and length of an arc of 1° of the parallel at Q

$$= \frac{2\pi \, QM}{360} = \frac{\pi}{180} \nu \sin c = \frac{\pi}{180} \frac{a \sin c}{(1-e^2\cos^2 c)^{\frac{1}{2}}}.$$

In fig. 2, using the same letters P and Q as there will be no confusion

let O be the centre of projection

P „ the position of the pole on the projection

Q „ the position of a point whose co-latitude is c

x „ the distance OP

y „ the angle of projection POp where Pp represents one degree of parallel.

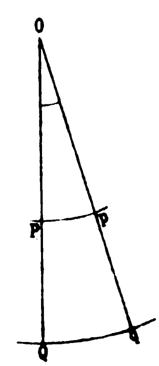


Fig. 2

Then the length of the arc of 1° of parallel at Q on the projection

$$= y\,(x + PQ) \times \text{circular measure of } 1°$$

$$= y \left\{ x + \int_0^c \frac{a(1-e^2)}{(1-e^2\cos^2 c)^{\frac{3}{2}}} \, dc \right\} \frac{\pi}{180}$$

Therefore the error of 1° of parallel at Q

$$= \frac{\pi}{180}\left[y \left\{ x + \int_0^c \frac{a(1-e^2)}{(1-e^2\cos^2 c)^{\frac{3}{2}}} dc \right\} - \frac{a \sin c}{(1-e^2\cos^2 c)^{\frac{1}{2}}} \right]$$

This is maximum with regard to c when

$$y \frac{a(1-e^2)}{(1-e^2\cos^2 c)^{\frac{3}{2}}} = \frac{a\cos c}{(1-e^2\cos^2 c)^{\frac{1}{2}}} - \frac{a e^2 \sin^2 c \cos c}{(1-e^2\cos^2 c)^{\frac{3}{2}}}$$

that is when

$$y(1-e^2) = \cos c\,(1 - e^2\cos^2 c - e^2 \sin^2 c)$$

$$= \cos c\,(1-e^2)$$

or when $y = \cos c.$

Now, neglecting powers of e above the fourth

we have $(1 - e^2\cos^2 c)^{-\frac{3}{2}} = 1 + \frac{3}{2}e^2\cos^2 c + \frac{15}{8}e^4\cos^4 c$

$$\text{also } \cos^2 c = \frac{1+\cos 2c}{2}$$

$$\cos^4 c = \tfrac{1}{4} + \tfrac{1}{2}\cos 2c + \tfrac{1}{8}(1+\cos 4c)$$

$$= \tfrac{3}{8} + \tfrac{1}{2}\cos 2c + \tfrac{1}{8}\cos 4c.$$

Therefore $(1 - e^2\cos^2 c)^{-\frac{3}{2}} = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + (\frac{3}{4}e^2 + \frac{15}{16}e^4)\cos 2c + \frac{15}{64}e^4\cos 4c$

and $\dfrac{1-e^2}{(1-e^2\cos^2 c)^{\frac{3}{2}}} = 1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 + (\frac{3}{4}e^2 + \frac{3}{16}e^4)\cos 2c + \frac{15}{64}e^4\cos 4c$

whence $\displaystyle\int_0^c \frac{1-e^2}{(1-e^2\cos^2 c)^{\frac{3}{2}}} dc = (1 - \tfrac{1}{4}e^2 - \tfrac{3}{64}e^4)\,c + (\tfrac{3}{8}e^2 + \tfrac{3}{32}e^4)\sin 2c + \tfrac{15}{256}e^4 \sin 4c.$

Using Everest's 1st set of constants $a = 20922932$ feet and $e^2 = 0{\cdot}006638$

we get

$$PQ = a\left\{{\cdot}998339\,c + {\cdot}002493 \sin 2c + {\cdot}000003 \sin 4c\right\}$$

so that $PQ_{82} = 1{\cdot}429478\,a$ and $PQ_{50} = 0{\cdot}873670\,a.$

Again

$$(1 - e^2\cos^2 c)^{-\frac{1}{2}} = 1 + \tfrac{1}{2}e^2\cos^2 c + \tfrac{3}{8}e^4\cos^4 c$$

$$= 1 + \tfrac{1}{4}e^2 + \tfrac{9}{64}e^4 + (\tfrac{1}{4}e^2 + \tfrac{3}{16}e^4)\cos 2c + \tfrac{3}{64}e^4\cos 4c$$

so that

$$\frac{\sin c}{(1 - e^2 \cos^2 c)^{\frac{1}{2}}} = (1 + \tfrac{1}{4} e^2 + \tfrac{9}{64} e^4) \sin c + (\tfrac{1}{8} e^2 + \tfrac{3}{32} e^4)(\sin 3c - \sin c)$$

$$+ \tfrac{3}{128} e^4 (\sin 5c - \sin 3c)$$

$$= (1 + \tfrac{1}{8} e^2 + \tfrac{3}{64} e^4) \sin c + (\tfrac{1}{8} e^2 + \tfrac{9}{128} e^4) \sin 3c + \tfrac{3}{128} e^4 \sin 5c$$

$$= 1{\cdot}000832 \sin c + {\cdot}000833 \sin 3c + {\cdot}000001 \sin 5c$$

therefore $QM_{82} = 0{\cdot}990332\, a$ and $QM_{50} = 0{\cdot}767097\, a$

The error of 1° of parallel in latitude 8° $= \frac{\pi}{180}\left\{y(x + 1{\cdot}429478\, a) - 0{\cdot}990332\, a\right\}$

" " " 40° $= \frac{\pi}{180}\left\{y(x + {\cdot}873670\, a) - 0{\cdot}767097\, a\right\}$

Since these two are to be equal,

we have $0{\cdot}555808\, y = 0{\cdot}223235$

so that $y = {\cdot}401640.$

The place where the error of parallel is maximum is given by

$$\cos c = y = {\cdot}401640$$

so that $c = 66° \; 19' \; 9''.$

We have therefore

$$PQ_c = 1{\cdot}157396\, a \quad \text{and} \quad QM_c = 0{\cdot}916288\, a$$

Therefore

the error of 1° of parallel where it is maximum

$$= -\frac{\pi}{180}\left\{y(x + 1{\cdot}157396\, a) - 0{\cdot}916288\, a\right\}.$$

Equating this to the error in latitude 40° we get

$$y(2x + 2{\cdot}031066\, a) = 1{\cdot}683385\, a$$

or $2x + 2{\cdot}031066\, a = 4{\cdot}191278\, a$

whence $x = 1{\cdot}080106\, a.$

The error of parallel vanishes when

$$y(x + PQ) - QM = 0$$

that is

$${\cdot}433814 + {\cdot}400973\, c - 1{\cdot}000832 \sin c + {\cdot}001001 \sin 2c$$
$$- {\cdot}000833 \sin 3c + {\cdot}000001 \sin 4c - {\cdot}000001 \sin 5c = 0$$

or $${\cdot}433814 - {\cdot}600357\, c + {\cdot}169229\, c^3 - {\cdot}009777\, c^5 + {\cdot}000546\, c^7 - {\cdot}000052\, c^9 = 0$$

This equation can be solved by any of the usual methods for the numerical solution of equations but it is easier to get the solution by trial.

Let $f(c)$ represent the left hand side

then putting	$c = 1$	we find	$f(c) = -\cdot 00660$
„	$c = \cdot 96$	„	$f(c) = -\cdot 00041$
„	$c = \cdot 95$	„	$f(c) = +\cdot 00134$

so that a root lies between ·95 and ·96 and on trial it appears that when $c = \cdot 9575$, $f(c) = +\cdot 00002$,

so that c is a little greater than ·9575 which corresponds to an angle of 54° 52′.

Similarly we find that if $c = 1\cdot 35$ $f(c) = -\cdot 00046$

if $c = 1\cdot 355$ $f(c) = +\cdot 00045$

and if $c = 1\cdot 3525$ $f(c) = -\cdot 00001$

so that $c = 1\cdot 3525$ satisfies the equation and this corresponds to an angle of 77° 30′.

The two parallels of no error are therefore those of latitudes 12° 30′ and 35° 8′.

Again the maximum error, *viz.*, that which occurs at the parallels of 8°, 36° 19′ 9″ and 40° is

$$\frac{\pi a}{180}(1\cdot 007949 - \cdot 990332) \text{ or } \cdot 017617\frac{\pi a}{180}$$

so that the percentage of error at these three parallels

$$\text{is at } 8^\circ = \frac{1\cdot 7617}{\cdot 990332} = 1\cdot 8$$

$$„\ 23^\circ\ 40'\ 51'' = \frac{1\cdot 7617}{\cdot 916288} = 1\cdot 9$$

$$„\ 40^\circ = \frac{1\cdot 7617}{\cdot 767097} = 2\cdot 3$$

Similarly the percentage of error in latitude 4° = 3·8

It only remains now to construct a table for plotting the sheets.

A reference to fig. 2 will show, if we consider PQ q p as any half sheet PQ being the central meridian, that

the co-ordinates of Q are 0 and 0

„ „ P „ PQ and 0

„ „ p „ OP versin O + PQ and OP sin O

„ „ q „ OQ versin O and OQ sin O.

The angle O being $2y^\circ$ or 48′ 11″·8 we get the following table :—

Rectangular Co-ordinates for plotting the Graticules of the Sheets, Scale 1 : 1,000,000.

Latitude		Meridians from Origin			
		0°		± 2°(East or West)	
		Meridian	Perpendicular	Meridian	Perpendicular
		Inches	*Inches*	*Inches*	*Inches*
SHEETS ...	40° ...	17·478	0	17·526	6·877
	36° ...	0	0	0·050	7·122
,, ...	36° ...	17·466	0	17·516	7·122
	32° ...	0	0	0·052	7·367
,, ...	32° ...	17·455	0	17·507	7·367
	28° ...	0	0	0·053	7·612
,, ...	28° ...	17·446	0	17·499	7·612
	24° ...	0	0	0·055	7·856
,, ...	24° ...	17·436	0	17·491	7·856
	20° ...	0	0	0·057	8·101
,, ...	20° ...	17·429	0	17·486	8·101
	16° ...	0	0	0·059	8·345
,, ...	16° ...	17·422	0	17·481	8·345
	12° ...	0	0	0·060	8·589
,, ...	12° ...	17·417	0	17·477	8·589
	8° ...	0	0	0·062	8·834
,, ...	8° ...	17·414	0	17·476	8·834
	4° ...	0	0	0·064	9·078

Directions for Plotting.—A point on the sheet is selected for the intersection of the central meridian and the lower parallel. Through this point a horizontal line is drawn across the sheet, and from the same point a second line perpendicular to the first: the second line will be the central meridian of the sheet. With these two lines as axes the intersections of the meridians and parallels are plotted from this table of rectangular co-ordinates in the usual way. As the sheet is symmetrical about its central meridian, the values for 2° are to be used for intersections both east and west of that meridian.

The exterior meridians and parallels are formed by joining these points by straight lines and the central parallel is obtained by bisecting the meridians and joining up the points.

ST. G. C. GORE, COLONEL, R.E.

Exaggerated Co-ordinates to be used only in plotting Sheets on Scale 12 miles = 1 inch for reduction to Scale 1 to 1,000,000.

REVISED PROJECTION

MAP OF INDIA AND ADJACENT COUNTRIES.

Rectangular Co-ordinates for plotting the Graticules of the Sheets, scale 12 miles = 1 inch.

Latitude	Meridians from Origin			
	0°		± 2° (East or West)	
	Meridian	Perpendicular	Meridian	Perpendicular
	Inches	*Inches*	*Inches*	*Inches*
SHEETS ... 40° ...	22·988	0	23·051	9·045
36° ...	0	0	0·066	9·367
" ... 36° ...	22·972	0	23·038	9·367
32° ...	0	0	0·068	9·689
" ... 32° ...	22·958	0	23·026	9·689
28° ...	0	0	0·070	10·011
" ... 28° ...	22·945	0	23·015	10·011
24° ...	0	0	0·072	10·333
" ... 24° ...	22·933	0	23·005	10·333
20° ...	0	0	0·075	10·654
" ... 20° ...	22·923	0	22·998	10·654
16° ...	0	0	0·077	10·976
" ... 16° ...	22·914	0	22·991	10·976
12° ...	0	0	0·079	11·297
" ... 12° ...	22·908	0	22·987	11·297
8° ...	0	0	0·081	11·618
" ... 8° ...	22·903	0	22·984	11·618
4° ...	0	0	0·084	11·939

Directions for Plotting.—A point on the sheet is selected for the intersection of the central meridian and the lower parallel. Through this point a horizontal line is drawn across the sheet, and from the same point a second line perpendicular to the first: the second line will be the central meridian of the sheet. With these two lines as axes the intersections of the meridians and parallels are plotted from this table of rectangular co-ordinates in the usual way. As the sheet is symmetrical about its central meridian, the values for 2° are to be used for intersections both east and west of that meridian.

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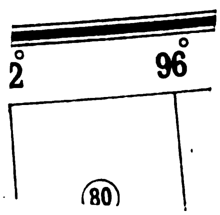
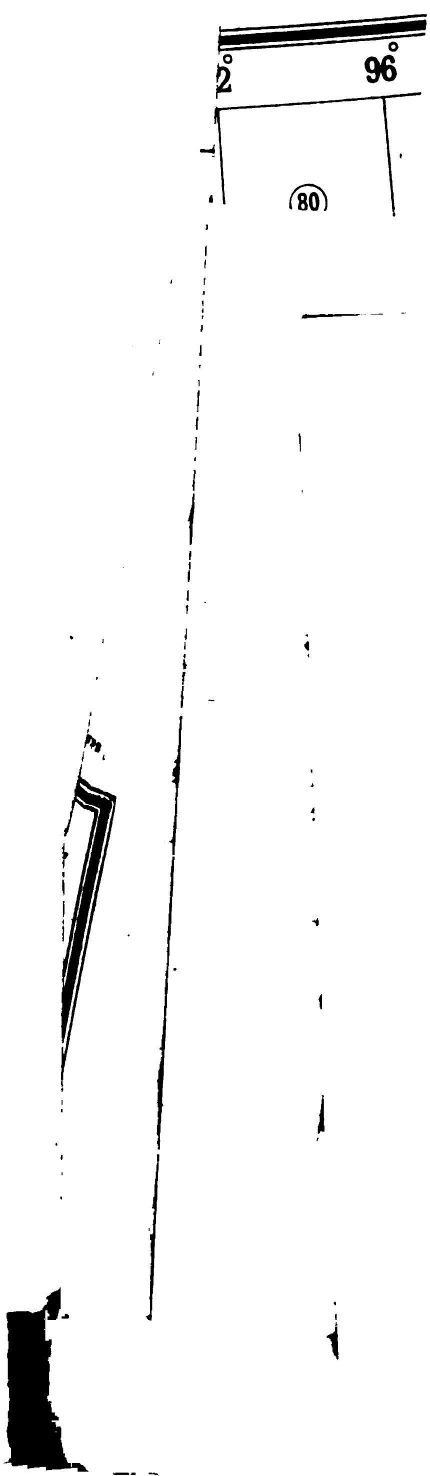

2°
96°
80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